AN INTRODUCTION ON THE UNIVERSAL APPLICATIONS OF HEXACHORDS IN SAXOPHONE PERFORMANCE AND TECHNIQUE

BACHELORARBEIT

zur Erlangung des Titels Bachelor of Arts (BA)
in der Studienrichtung ..................JAZZ SAXOPHON
im Studiengang ..........................JAZZ
der Fakultät .................................MUSIK

der Musik und Kunst Privatuniversität der Stadt Wien

eingereicht von

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Introduction

This paper is written as the beginning of my research into creating a new method approach with the saxophone, however it can be applied to any melodic instrument as seen fit. This work is meant as a handbook for understanding the Hexachord System, and how to incorporate it in any modern music study. I will not attempt to say there should only be one accepted practice of music theory and instrumental pedagogy. My research in this paper is merely a reflection of my time studying in Vienna and the tools I will carry on with me. That being said, one of the reasons I am writing this paper is to offer seeking minds a chance to see their music (what they are studying, composing, or exploring) through a new lens of Hexachords and the Hexachord System. It is my hope and goal that by the end of this paper, each reader will have a solid understanding of the technical, theoretical, and improvisational concepts of the hexachord, as well as its significance and importance in music’s development.

The majority of the concepts discussed in this work come from my time studying with Walter Ernst Haberl at the Musik und Kunst Universität der Stadt Wien, as well as in private conversations between the two of us. My contributions to the Hexachord System’s development are within the reach of the Exercises presented; much of the content is not new, however my melodies and approach to the Saxophone are my own.

As this paper is written as both the groundwork for a teaching book, as well as the scientific research behind it, the reader should be aware of different styles of explanation and voice throughout the text. Much of the content in this work will be directly used in further publication in either part or full, and this research will provide the factual and formal background for such situations.

I believe that every person, ‘musical’ or not, is aware of Hexachords, but may not always know what they are. My thought is, with this work, that I can better bridge the gap from someone saying, ‘Aha! I know that melody’ to ‘Aha! I know what that is.’ In my first moments with the Hexachord System and understanding its function in music, the “Eureka!” moments were endless. My compositions changed, as did my basic approach to music. I was finally at home with the melodies, the sounds I knew in my heart, and the applicable theory to
go with it. However, as a saxophone student, I felt a lack of direct understanding on my instrument. All of the original publications regarding the Hexachord System were aimed for the piano, so as I began studying the System in depth, I was finding myself focusing more and more on the piano. There is no better instrument to visually understand what you are playing, and that is what led me to this book. Many of the music greats used Hexachords in their music, perhaps unknowingly. As a Saxophonist, a great example is Charlie Parker. His influence in the development of American Jazz and Bebop was fueled by the knowledge of Bach and the Classical Greats. Parker’s musical language works flawlessly in the Hexachord System, as it is heavily influenced by music written in this fashion. However, the over-analysis of his work (applicable to both Parker and Bach) postmortem has led to far too many “overworked” approaches to music, both in the Jazz and Classical education realms. I needed something for myself; both to better understand an approach on the saxophone, and something to pass on to the generation after me. Therefore, I decided to create this book as a teaching guide and learning companion, in the style that I would have loved to have when I first brought this element of music into my life.

We will explore the Hexachord System in a few ways throughout the book, but do not let these big words scare you. This old system was taught to all the great composers until it was wiped away from the history books and sadly forgotten in most music education classes. In this work we will cover the basics — I do not want to overdo it! It is important that every music student has their own time to understand and process the material presented in their own way. The exercises I chose for this work should provide enough learning material without being too much to handle in one sitting. However, these exercises should still have enough weight in their content that they can be worked with for weeks, months, or even years.

The two main parts of this paper consists first of an overview of the Hexachord’s basic history, a description of its structure, and a run-through of how the Hexachord System works. The explanatory preface will act as the first presentation of the research that led to the content in the workbook. After the written explanation, readers will find an attached copy of the workbook, where they may go through the exercises and apply the concepts to their instrument as they see fit. The workbook will explain each individual exercise while guiding the reader through the content as a comfortable dialogue appropriate for any age. As the
This work uses Hexachords and their harmonic and melodic functions as the fundamental element of music theory. The Hexachord was first seen in publication in the *Micrologus*, the featured work from Guido d’Arezzo. Its importance in music education, composition, and performance extended well into the mid-19th century, even though it was no longer publicized after the mid-1700’s. The way Hexachords function in music is very simplistic, however when beginning to understand them in practice, it often requires a bit of re-learning what we know. Revered or feared by many music educators of today’s practice, the Hexachord System offers the user flexibility and versatility when composing, improvising, or studying music, as it did with the grand masters of centuries past.

### 1 GUIDELINES OF THE HEXACHORD SYSTEM

A Hexachord is a series of six consecutive notes grouped within the interval of a 6th. Any combination of notes within this interval can be named a Hexachord, creating either a six-tone scale or a set of two contiguous triad pairs. Each Hexachord has unique harmonic and melodic properties, as the combination of its notes allows it to be applied to various keys, progressions, and passages at any time.

#### 1.1 A History of the Hexachord System

The Hexachord System we will be discussing first appeared in the early 11th century throughout medieval music. This six-note shape found itself perfectly representing natural barriers of the human singing voice and its structure containing one tonality at a time. Starting with its earliest forms, the Hexachord System became the foundation for all written music because of its balanced and uncomplicated nature.
1.1.1 Guido d’Arezzo

Guido d’Arezzo’s original Hexachord model used only one Hexachord (C-D-E-F-G-A) as the primary tone-room for all music. Later a secondary Hexachord, or Hexachords, would be added. D’Arezzo’s multi Hexachord model still used the C Major Hexachord as the base, but this time the extension of the G Hexachord (G-A-B-C-D-E) below and F Hexachord (F-G-A-Bb-C-D) are connected. The idea behind this system was that when a vocal melody needed to extend past the normal hexachord range, to a B or Bb, choral singers would do so by shifting Hexachords on a new created half-tone axis, either above (A-Bb) or below (C-B). The human singing voice cannot naturally sing a 7th, however it can easily sing a 4th.

![Hexachord Diagram](image)

**Music Example 1**

Here, a full one-octave range is spaced into three Hexachords, with the C Hexachord surrounded by its “durum/hard” B from the G Hexachord below, and its “molle/soft” Bb above. This way, the voice would shift its focal point when reaching either a Major or Minor “7th,” only to be singing a comfortable “Fa” in a natural way. In the example above, the two focal shifts begin on either the F going up, or the E going down. The descriptions of these hard and soft B notes led to the later markings representing Naturals and Flats as accidentals, due to the style of handwriting in the time. By the early Baroque Era, other accidental notes, like F# or C#, were added into use by repositioning the mi note of each Hexachord to any altered note.¹

1.1.2 The Beginning of the End: Buttstett vs. Matteson

At the height of the Baroque Era, all great performers and composers were not only aware of the main three hexachords, but also the permutations and alternate forms of the Hexachords where the position of the half-tone (or two) were placed.\(^2\) To strengthen this school of thought, Johan Heinrich Buttstett, J.S. Bach’s uncle, published a treatise in 1716 titled, “Ut, Re, Mi, Fa, Sol, La est Tota Musica et Harmonia Aeterna.” In his publication, Buttstett used a Star of David (Jewish Star) as the symbol of the Hexachord System. The Star of David consists of two triangles, which relate directly to the two triads present in one Hexachord (C-E-G, and D-F-A) respectively. In addition, the three small flames inside the star represent the inner Trichord structure inside of each Hexachord, amongst other things.\(^3\)

The Hexachord System, however, would meet its public demise around this time as well. Buttstett’s major rival, Johann Matteson, constantly published articles and material disputing “the old system,” and his work Das Neu-eröffnete Orchestere (1713) was the first open criticism of the Hexachord System as well as the first example of the Common 24 Major and Minor Scale music theory. Treatise after treatise from both sides, Buttstett continuously explained and proved in his work how Matteson’s system was simply a small portion of the Hexachord System; the 24 Scale Theory is easily incorporated by the fusing of two Hexachords with each other. But Matteson was adamant that the old system needed to be left in the past. Buttstett heavily feared that Matteson’s work would destroy an entire system of music that functioned flawlessly for hundreds of years, and alas Matteson’s swift public maneuvering left the former traditional System defenseless.

The loss of the battle against Matteson meant a public end to the Hexachord System, not only being taught in schools, but also from the history books. There is very little correct mention of the Hexachord System or the uses of Hexachords themselves in modern works, musical or not. The rise of Music Education programs has created a vast source of learning material, however there are still gaps in understanding when an analysis of music written in the

\(^2\) Walther, Johann Gottfried (Practica der Musikalischen Composition Weimar 1708); Buttstett, Johann Heinrich, Erfurt 1716; Agricola, Johann, Tosi, Schiebe, Johann Adolf, 1772. At this time, musicians versed in the Hexachord System began experimenting with the Improvisational potential of the element, creating new sounds commonly used in many famous compositions of the era. Some of the most well known Hexachords are in fact permutations of the original Guidonian example.

\(^3\) Because of its Base-3 figure, there are many pieces of the Hexachord System built on 3, or multiples thereof, including Tetrachords and smaller structures.
Hexachord System appears. I can recall that when discussing Mozart or Bach in the American High School’s highest national theory course, our teacher told us that “the Masters are an exception; they were so good they could do whatever they wanted.” In time, I learned this was simply untrue.

Although most music pupils throughout the 18th and 19th century would not be learning in the Hexachord System style, many of the young masters continued their practice in the old system. In general, the Hexachord System provides a flexible path of little resistance when composing. The connectivity between structures allows fluidity in the statement of themes, their variations, and harmonic expansions as well. Although in the 18th Century, the new (8 Tone) system was en-vogue; it did not stop the composers of this time from using a tried and true method in their work. Further analysis in the works of any great composer until the Second Viennese School\(^4\) show a direct use of the Hexachord System throughout. The Hexachord System is truly the forgotten secret of *Wiener Klassik*.

### 1.1.3 Trichords, and Mozart’s Introduction to the Hexachord System

The inner Trichord shape inside of each Hexachord allows the user to perform and practice the transitions between tonalities smoothly. When Wolfgang Amadeus Mozart was a young child, he traveled with his father, Leopold, all throughout Europe, with the opportunity to learn from and study with each city’s grand Masters. It was during his stay in London that he first became acquainted with Johann Christian Bach and a connection was formed. As Mozart arrived at J.C. Bach’s home for lessons, the first thing Bach corrected in Mozart’s playing was his finger position.\(^6\) By repositioning young Mozart’s first three fingers on the first three notes of a Hexachord (Thumb, Index, Middle on C-D-E), Bach could properly begin to teach the Hexachord System.

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\(^4\) The AP (Advanced Placement) Music Theory course is the highest offered level of music history in the American School system. The Advanced Placement courses are designed to offer High School students a College level class in the specific subject, sometimes allowing the student to “Place Out” of the course when attending university.

\(^5\) The Second Viennese School refers to Arnold Schönberg and his composition students in the early 20th Century.

\(^6\) As the quote from Nannerl Mozart states, “*Herr Johann Christian Bach, the Queen’s teacher, took the son between his legs, the former played a few bars, and the other continued, and in this way they played a whole sonata, and someone not seeing it would have thought that only one man was playing.*” It was in these lessons that Mozart learned intensive keyboard skills, as his father would later remark after their time in England how much progress the young Wolfgang made.
The Hexachord System did not end with Bach or Mozart. Many artists continued to base their compositions on this System until the era of the Second Viennese School. In fact, the entirety of the “First” Viennese School and the style it represents is all attributed to the Hexachord System and the way it works. Later, the Hexachord System would find its new life in many improvised music genres outside of classical; it would become an unspoken and integral part of Charlie Parker’s Bebop language on the Saxophone.  

1.2 Theory

Johann Sebastian Bach was the premiere example of the Hexachord System’s master. His compositions are clean examples and mathematical proofs of the Hexachord System, especially those that cannot be described by common music theory practice. In current traditional “Tonal Harmony” theory, each individual note is examined vertically, not horizontally. In Hexachord Theory, the focus is placed on the latter.

When describing music theory within the Hexachord System, it’s important to discuss how it was originally taught. The foundation is simple: Melodies are played within one Hexachord range, and cadential moments, or harmonic progressions, occur when two Hexachords are combined, extending the range of melody and harmony to achieve the desired result. The first two basic approaches are Dominant and Plagal, relating to the position of the secondary Hexachord in relation to its Primary. A Dominant Approach occurs from below as a V-I, much like common practice’s Perfect Cadence. The Plagal Approach, like the Plagal Cadence, is a IV-I from above.

In practice, this relates to the two Servum possibilities per Primary Hexachord as well. As with our C Hexachord, the Dominant Servum relates to the connection on the first trichord (G-A-B) in G-A-B-C-D-E, whereas the Plagal would relate to the F-G-A-Bb-C-D, connecting to the second trichord (F-G-A).

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7 Charlie Parker was in many ways the father of a new type of Saxophone performance — his fast-paced, flowing lines and their structure were beautifully new in their time, and are heavily used in current Jazz Education programs as the base model of improvisational techniques.
Much of the Hexachord System is focused on the Primary Hexachord, simply because so much can be expressed within one range.

It’s best in this situation, however, to start with a strict “Ambitus” and stay within one Hexachord, as d’Arezzo first taught as well. As the musician studies one Hexachord thoroughly, he/she will find that the possibilities within one Hexachord range may run out at some time. By practicing one Hexachord, its melodic, harmonic, and contrapuntal parts enable the musician to see the next practical step: reaching out to the first Secondary Hexachord, in best practice placed below.

1.2.1 Major and Minor Hexachords

It is important here to state clearly what Major and Minor means in the Hexachord System. A Hexachord’s Major or Minor quality comes from its boundary interval of a 6th. A Major Hexachord is one with a Major 6th as the interval, whereas a Minor Hexachord is one with a Minor 6th. This basic principle simplifies the necessity for hundreds of different types of extending scales with different harmonic properties! The user may define the notes in their hexachord(s) as desired. Even with altered tones that may create common ‘minor’ sounds, for example a flat 2nd, 3rd or 5th, a Hexachord can still be seen as Major as long as its outer boundaries remain the Major 6th interval.

In much of W.A. Mozart’s work, his Hexachords can be seen as such. The examples below show that even the Whole-Tone Scale is Hexachordal. Mozart calls it an Augmented Hexachord, and in many ways, that’s exactly what it is.8

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8 The following examples are from Grandjean, Wolfgang: Mozart als Theoretiker der Harmonielehre & seine “Kurzgefaßte Generalbass-Schule” Olms Verlag 2006
1.2.2 Added Servum: The Secondary Hexachord

Once the Secondary Hexachord (Servum) is in position, it brings the first cadential possibilities to any melody or harmonic progression. Now the musician can express the Subdominant or Dominant characteristics of the newfound tonality, whereas previously within one Hexachord, the tonality was kept inside its first barrier. Now that the first 7th interval of a complete mode can be heard, all possible 7ths within the two-Hexachord room can be expressed as well.

The main use and function of the Secondary Hexachord is to briefly shadow the complimentary tonality of the Primary before returning. When a melody stays too long in one Primary Hexachord, the sound becomes monotonous and loses its momentum. Leaning on the Secondary allows the melodic motion to continue throughout the passage, balancing the tonality of a melody upon arrival at its origin, or allowing it to travel further into a new tone-room.
An extended use of the Secondary Hexachord is the type of chromatic connection that can be formed when moving from one Hexachord to another. In Charlie Parker’s Bebop lines, his melodies can often be confined to a Hexachordal shape, maintaining the popular Major 6th chord sound. When moving through a 2-5-1 progression or expressing in longer chromatic lines, the same principle is used. Leaning on the Secondary Hexachord allows the 7th to be played; bringing the dominant difference to the sound that is inherent to jazz. Often, however, both the Major and Minor 7th (both variations of the Servum Subsemitonium) are played to keep the melody in phrased time and rhythm.

Contrary to common teachings today, a Bach/Mozart scale would go as follows: C major = (B)-C-D-E-F-G-A, instead of C-D-E-F-G-A-B-C. The name of this scale is the “Schemata Modorum” (Heinichen: Dresden 1728). It is a Primary Hexachord with the Subsemitonium modi, or as commonly called the “leading tone” B.

1.2.3 Movable “Do” and Other Options

There is a common misconception when discussing one Hexachord and the tonality it creates. When the notes C-D-E-F-G-A are presented and played in their closed room Ambitus, the sound heard is closely related to C Ionian. However, this Primary Hexachord has its own Secondary and Tertiary Hexachords, and their own unique tone-rooms. The misconception appears when approaching the other notes in this one Hexachord! Just because there is a D, or Re, in a C Major Hexachord, does not mean the D must sound as D Dorian. The D in this Hexachord represents that its own Do-Based tone-room can be used, allowing the user to play the D Minor/A Dominant sounds related to C, as well as its own more distant modes, including D Major sounds, Lydian or Phrygian. In its essence, any note present can be the base of its own Hexachord, Major or Minor, Augmented, or Diminished!

This principle continues to hold its ground throughout the entire Hexachord System. Any note present represents its own possibilities, therefore allowing chromatic movement between

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9 The Generalbass book from Johann David Heinichen was a personal undertaking of J.S. Bach; he led the publishing and further distribution of the work to the Leipzig area.

10 A perfect example of the Schemata Modorum can be heard in J.S. Bach’s “Brangenburg Concerto Nr. 3, Second Movement, Allegro.” This sound can also be both heard and seen in “Practice Exercise 3” in Part 3 of this work.
notes at any given time. The success of the Hexachord System lies with the user’s creativity and personal musical expression and journey. All music should not be kept inside the same box.

[Page Break for end of Part 1]
2 WORKBOOK

Welcome!

This notebook will serve as the first written version of your Hexachord System introduction work/methodology for melodic instruments. In this book we will cover a few different types of Hexachords, how they function in the Hexachord System, and a series of exercises and Etudes for you to begin adapting this system of music into your own.

My goal is to layout enough content and context for musicians of all ages and levels to appreciate and understand the Hexachord System in a way common music education does not. This work is not meant to attack any form of theory. It is simply presenting a method used by all of the Master composers that we know and continue to discuss and perform today. There may be an element of relearning what you know in this new lens, but I hope it will be exciting and pleasant! Music should be exactly that.

2.1 From the Beginning... Our First Hexachord!

\[ \text{Music Example 3} \]

The notes above, as written, form a Hexachord. A Hexachord is composed of six consecutive music notes inside the interval of a 6th. Most Hexachords contain only one Half Tone, as shown in this case between the A# and B, however later we will learn that Hexachords can have more. I chose the F# Major Hexachord to clearly define the interval in a nice representation along the staff. (A side reason for choosing a F# Major Hexachord is to pay tribute to the great Irving Berlin. Of the thousands of songs he wrote, he could only play and compose in the key of F#.)

People hear Hexachords differently, which is why I will describe them Symbolically, Harmonically, and Melodically in this work. However, I will give extra attention to Melodic functions to avoid confusion of the word “chord” and to keep this work Saxophone/Melodic Instrument related.
The first written mention of Hexachords and the Hexachord System appeared in the 11th century work from Guido d’Arezzo titled, “Micrologus“ — a sum of all music at the time. I will go into d’Arezzo’s model later. For now, I want to explain more about Hexachords in different terms before explaining the Hexachord System in full.

2.2 Children’s Songs: Our First Contact with Hexachords

That’s correct! For most of us, our first contact with Hexachords came from children’s songs! Nearly all children’s songs are based in Hexachords with the majority consisting of only one Hexachord. One of the primary reasons of the Hexachord being the foundation of medieval music’s notated compositions is due to the fact that the human singing voice’s natural range is a Sixth! From childhood play songs to the holy chants of many religious prayers, their melodies are within this natural range for a reason.

Music Example 4

It seems that almost everyone learns “Frère Jacques” at a young age... I know I did! What about this Mozart melody below?
Music Example 5

If you look carefully, you’ll see that Hexachords are everywhere. I will bring more song examples shortly, but first I’d like to discuss another important topic...

2.3 The Hexachord Symbol and its Meaning

This is the symbol for a Hexachord, as published in Johann Heinrich Buttstett’s 1716 “Ut, Re, Mi, Fa, Sol, La es Tota Musica et Harmonia AEterna.” The design of the Star of David fits perfectly to Hexachords, and here’s why.

In music, basic harmonic structures containing three notes played consecutively are called Triads. These Triads consist of two intervals of a 3rd stacked on top of each other. Numerically, a Triad starting from the root note of a scale, or “Do,” will have the notes 1, 3, 5, or “Do, Mi, Sol.” The type of 3rd interval in a Triad defines its sound, either being major or minor (the latter with a flat-3rd instead of a natural 3rd).
The two Triads consecutively laced in a Hexachord help define its harmonic function. The smaller figures of 3 will help solidify the Balance of a Hexachord throughout. For example, can you see how the various pieces and their mirrored inversions make them essential parts of a Hexachord’s Harmonic and Melodic functions?

Music Example 6

2.4 “Mirror” System and Trichord Breakdown

The inner shapes of a Hexachord are triangular in almost every aspect. The way each angle of a Hexachord intertwines with other Hexachords is truly remarkable. Each tonality switch becomes fluid and active instead of passive juxtapositions. The smaller shapes shown next to the Triads are called Trichords. A Trichord, similar to a Hexachord, is named due to the number of notes present. It is not played consecutively like a Triad or other Major/Minor Chord. Trichords correspond to three contiguous note groupings, like 1-2-3, or 4-5-6. The Trichord’s function in the Hexachord System is crucial to the melodic properties of music, and we will continue to discuss these throughout the entire work.

As we move forward, the “mirror” concept will return. Many of the Hexachord System’s expansions and developments come from mirroring techniques. See if you can find the “mirror” in the next example below.
2.5 Hexachordal Analysis of “You Are My Sunshine”

The first part of the melody becomes mirrored when its response melody completes its thought. The “lean-to” balance in Melody comes from the inner trichord connections working together to build tonality or key centers.

Looking again at “You Are My Sunshine,” we can both hear and see the tonality changes that occur. The notes we have in the song are most closely related to the G Major Hexachord, but the melody may leave the boundaries. Remember, this does NOT break the rules!

Yes, the song may only use the notes present in the G Hexachord (1), but look at the intervals of the melody (2). It has the same boundaries as (3) The D Major Hexachord.

The response to the first part of the melody (4) is crucial in its development. Why? Because the inner Trichord (G-A-B) is pronounced clearly, centering the tonality around the G Hexachord. In most songs, repetition of one Trichord is used to temporarily focus on a new key center.
The moving Key Center, now focused on G, stays active, as the next Trichord is introduced: C-D-E, with the Trichord connected to G-A-B. In full, the three Trichord Ambitus looks as follows.

Music Example 9

3 THE FIRST EXERCISES!

Now that you have a basic understanding of the groundwork in Hexachords and their structure, it’s time to start bringing them into your playing. The first sets of exercises contain one Hexachord apiece. Each exercise will be written out in all 12 common major and minor keys for beginning musicians to become more comfortable reading in different keys and translating the notes to the instrument. The goal of these first exercises is to deepen your ear’s harmonic understanding of a single hexachord range. By playing and practicing the basic melodic line of a Hexachord, you will create the aural paths necessary to complete this work.

The first exercise will present the Major and Minor Hexachords in a strict Ambitus and will look as follows:

Music Example 10

3.1 Practice Exercise 1 with Ambitus

In the mono-Hexachordal exercises, I have left out the key signatures and placed the accidentals on each line instead. The single Hexachord Ambitus does not always reach the
required notes to “fit” common key signatures, so I will focus on the individual sound of each figure.

The first exercise is to be played in a Straight 8th, classical feel. DO NOT RUSH! Allow your fingers to adjust to the pace. There is no need to start too fast. Again, the 16th Notes DO NOT need to be fast! I chose to start the first two exercises with one straight 8th and one swing rhythms, to appeal to both pupils of the Jazz and classical pedagogues.

[Page Break for Practice Example 1]
Hexachord Practice 1, Straight 8ths, "Major"
3.2 Practice Exercise 2 Overview

In the first exercise, did you notice where the inflicted note is? The rhythmic displacement of Exercise 1 should accent the moving figures nicely, as well as keep the time steady.

This next exercise, Example 2, will approach the same Hexachord but in Swing 8th feel, allowing the practices to feel a different type of line when playing the same Hexachord.

Music Example 11

This exercise can be played in two ways: either played as above or with an added chromatic passing tone instead of the jump, as shown here:

Music Example 12

The flexible inner-tone room of the Hexachord allows for chromatic “Accenti” to be used at any point without altering the original tonality of the Hexachord. As you practice this next exercise, try out added chromatic passing notes! You may find new and exciting ways to add motion to the exercise that are unique and enjoyable to you.

(I have chosen not to include the chromatic version of exercise 2 on the next page to keep consistency with other parts of this book. As excited as I am to share as much as I know, I feel a sense of patience on my part will be helpful in maintaining proper order.)

Example Exercise 2 can be found on the next page.
Hexachord Practice 2, Swing 8ths "Major"
Hexachord Practice 2, Swing 8ths "Minor"
3.3 The Servum

The Servum is the name for a Secondary Hexachord that complements its Primary in either direction or key. It can be Major, Minor, or any auxiliary tonality shape. The key to any Servum lies in how it connects to its Primary. The use of the “fa-mi” bridge or “Lean-To” concept comes in handy here. Take a look at this example below:

Music Example 13

This small melody uses one Hexachord, [1] to introduce the theme. It isn’t until the 6th Measure, where small figure [2] is played, bringing the melody to its conclusion. This happens because of the Quick Borrow/Lean on the Servum Hexachord [3] which presents the G# next to its goal note, A.

Music Example 14

The only different/unshared note between the E Hexachord and the A is the G#, and the half step between these two notes is the strongest connecting point. These two Hexachords, when added together, form the Hypo-Ionian sound that we know by ear from pop music and all other genres.

Music Example 15
When placed together in Ambitus Form above, a clean example of running and attaching Trichords is shown. Like in the theme presented before, the two Trichord “leans” are in between the 1st and 2nd bars, as well as bar 6...

Music Example 16

3.3.1 One Note Servum

More often than not, instead of performing the entirety of the Servum’s boundaries, only a One-Note Servum will be used. From a religious perspective, my time learning many Hebrew chants of prayer made sense upon seeing the historical connection to the Hexachord. Interestingly enough, in all of my background research into Jewish music history, dating back to King David, the results were surprisingly eight tone based! It was said that David’s Lyre had 8 strings, yet the music we hear and sing is based on six? My thoughts are as such: the Hexachord is played in the center (notes 2-7), with one note acting as a barrier from either side. This relates exactly to Guido d’Arezzo’s model! (H) C-D-E-F-G-A (B) with alterations as seen fit: A perfect translation to the Hexachord System.

I know I have not yet described Bebop scales in depth, nor have I truly dived into Hexachordal Harmonic Properties. The brilliance of Hexachords is their multi-purpose usefulness, with each Hexachordal Structure being ready for any use at any time. There can be beautiful Harmonies made from the combination of the two triads— whether major/major, minor/major, major/minor, minor/minor, diminished, etc. That said, each combination of triads can be played a whole or half step away from its neighbor, creating very unique scale/chord options.

I think that there is merit to 8 tone scales - please don’t think that I do not! There are situations where longer scales are intended for a specific use. I’ve come to really admire
seeing scales as I have shown them above, and will also below. Especially when focusing of the Hexachords present, and their lean-to neighbors – it is crucial to centuries worth of music.

3.3.2 Why the Servum is Essential: “Proof in the Minor Pudding”

In a common Moll/Minor “scale,” what do we do with the 7th? Why are there three types of minor scales, each with a different 7th that feels out of place? That’s because it is. Look at this model of how the 8 Tone minor is built by hexachords.

![D minor scale and H.S. comparison]

Music Example 17

In this D Minor scale, the uncertain 7th, in this case the C#, is not clearly described in common Tonal Harmony. In most educational settings, the student will be taught that there are Natural, Harmonic, and Melodic Minors, each with a different 7th alterations, as well as different ways to perform and practice them. By viewing the sound as a Hexachord Minor, the choice is open to whichever type of auxiliary 7th seen fit! It can be the C# from a Hexachord Minor starting on A, or a C Natural as well! If, for example, the musician wants for further alter the sound, the option to even take a Bb Diminished Hexachord is open as well – any connecting possibility is allowed. The system and mathematics are the same; only the ear of the musician changes the meaning, direction, or alterations.

3.4 The Fa-Mi Bridge:

Throughout the work I have and will continue to talk about the “Mi-Fa” or “Fa-Mi” bridge. This term was coined by J.S. Bach as his formula\(^\text{11}\), as shown in this proof of a Major Scale to Hexachords below.

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\(^{11}\) As seen in the J.S. Bach publication, “\textit{Fa Mi et Mi Fa est Tota Musica},” Leipzig 1749.
Music Example 18

What Bach describes here is the connection between two Hexachords. The first “Fa,” C, jumps to a different “Mi,” E, instead of its own Mi, B. The C-Fa relates to the G major Hexachord, here being used as a Secondary Servum below the Primary Hexachord, or the note we want to call Do, C. By thinking Fa-Mi-Fa-Mi our brain will see the Hexachords fused together around the “home note” C. Similar to the D Moll/Minor scale proof in the Hexachord System, the similar concept is applied to common Major sounds.

3.5 Mozart and Finger Position (Mozart/Bach Piano Lesson)

There is a unique story about W.A. Mozart and his time with J.C. Bach where Bach corrected Mozart’s finger position on the piano, allowing Mozart to fully absorb the Hexachord System at a young age. Amongst the many influences J.C. Bach had on Mozart, the lesson of finger placement/position is important for any musician, especially those new to the Hexachord System. Expanded from Kirnberger’s transcriptions of J.S. Bach’s teachings, the basics of this lesson are as follows:12

In one Hexachord, and overall in the Hexachord System, only THREE FINGERS should be used when practicing, improvising on, and performing these figures.

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12 In Kimberger’s Grundsätze des Generalbasses, he states, as translated: “make all students aware of J.S. Bach’s found rule: in any possible case, before and after the Leading tone (Semitonium Modi), the thumb should be used to play the note that falls on the half or whole tone.”
On the keyboard, the first three fingers -- Thumb (1), Index (2), and Middle (3) -- are used. As Mozart was taught, the three fingers connect to the Trichords - they act in accordance with each other.

![Music Example 19](image)

By keeping the thumb and 3rd finger on the Trichord connections, the “Mi-Fa” principle of the Hexachord system is easily played.

Relating back to the Servum, this finger position allows swift transitions in between Hexachords without breaking their form or function.

As you practice Hexachords on the piano, always remember the Mozart/Bach secret. It is the forgotten key in Performance! (You may even find this fingering method can be used on most pieces by these composers as well.)

Just a thought: It does not require much searching nor learning to receive the same lessons of the grand masters. Your own answers or successes depend on your creations inspired by this knowledge.

### 3.6 Exercise 3

Try out the fingering exercise on the piano (if you are able to) with this next exercise you will find on the following page, Exercise 3: Hexachord with Natural Servum Below.

... Remember... 16th Notes up ahead — Don’t start Fast! Slow, slow, slow.
Hexachord Practice 3, Straight 8ths, "Major"
Hexachord Practice 3, Straight 8ths, "Minor"
3.7 Variations in Hexachord Types

There is a flexibility in each Servum that is open to unique performance options as well. Every Servum in the Hexachord System can be altered harmonically however seen fit. For example, in Exercise 3, Minor Variation, the Servum Trichord is first played in Natural, however shown as both in the Ambitus:

Music Example 20

The option of using accidentals for flexibility gives the user freedom to choose which Minor sound is desired upon use. In the example above, the G Minor Hexachord can contain the following accidentals while remaining the Servum:

Music Example 21

Technically, all inner notes can be altered, however the 4th (Fa) is crucial in deciding the type of tonality created. It is smart to avoid altering the 4th too much in minor; it is an easier process in Major (Ionian $\rightarrow$ Lydian).

Music Example 22
Here is a gentle reminder: In the Hexachord System, Major and Minor Hexachords are defined by their outer interval. Any Hexachord with a Major 6th as its boundary interval is considered Major. The same principle applies to a Minor Hexachord with its Minor 6th...

3.8 FURTHER CHROMATISCISM

In addition to complementing the Primary Hexachord with its Servum below, a Servum may also be placed above its Primary. Like our first exercise with a Servum, the connection between Hexachords still occurs at the Half Tone. Now, however, we will be reaching into the Dominant/Mixolydian Sound, whereas before we were still in a more Ionian feel.

This is a good place to mention one of the jazz applications for Hexachords and the Hexachord System. This next exercise will show how the commonly used Dominant 7th in blues or jazz settings functions dually as the 4th of a secondary Hexachord.

3.8.1 THE D’AREZZO MODEL, and BEBOP

Guido d’Arezzo’s original Hexachord Model kept all written music in a 3 Hexachord range. As written music was still forming its “normal,” this system was widely accepted for hundreds of years (although the musical complexity continued as the foundation remained).

It starts as such. All melodies are written in the C Hexachord Range. If a note (Bb or B/H) is needed, the singer shifts Hexachord Centers, so a comfortably reached “Fa” is sung, not a dissonant “Ti.”

Music Example 23
The two Hexachords that can be added are called the Durum and Molle. The Durum represents a Hard H (or B Natural); the Molle represents a Soft B Flat. In time... H —→ (natural), b —→ (flat)

D’Arrezzo’s model is important to bring in this work to Example 4, where the second Servum is attached above its Primary. As shown in this model below, the Secondary Servum brings the user into the Dominant sound we know.

Music Example 24:

... As for Jazz? How does a Bebop Scale work?

Music Example 25
That’s How!

3.8.2 EMPLOY & ERASE, or Indirect Resolution

The Employ & Erase concept, as coined by Walter Ernst Haberl, confronts the ongoing argument of Chromaticism in Classical Music. Contrary to popular belief, a chromatic passage or sudden uses of chromatic alterations on passing tones are a simple part of the Hexachord System’s linear resolution possibilities. It works like the Lean-To system for a Secondary Hexachord, however this is the quick two-step version when staying inside of one Hexachord range. A chromatic alteration is placed in a passage, however it is then erased upon its next use, without altering the Hexachord itself.

13 In the Jazz/Arranging realm, this concept is known as Indirect Resolution.
For example, while we are in the key of C Major, common music theory practice dictates that our only available notes are C-D-E-F-G-A-B-C. By now, after having gone through this material, I hope you will ask, but what about C#, F#, or others? Can’t they belong too? The answer is yes. You were correct.

To interject quickly: If an employed note is played twice without being erased, it usually means the composer is moving toward a new tonality/hexachord range.

One of music analysis’s greatest flaws is the lack of looking at music as a whole, rather focusing only on vertical structures. The Hexachord System allows musicians to flow from key to key in motion – that is the biggest principle by far! As stated earlier, any note inside of a hexachord can be altered or used without necessarily changing the Hexachord! It is only the outer interval that defines its function, Major or Minor. The user decides the inner voices and color as seen fit.¹⁴

³.⁹ FURTHER THOUGHTS

When I first wrote the initial Hexachord exercises, I thought that the content would be enough to show how the Hexachord System works. However after some time I found myself at a

¹⁴ The Employ & Erase concept will be shown further in Etude No. 7.
standstill. As this work is about the capabilities of the Hexachord System with MELODIC instruments, I decided it would be a better way to put their use to the test by writing Etude-based exercise melodies instead of a repeated system of small block exercises in all 12 keys repeated in common Major and Minor modes.

One of the largest misconceptions of Hexachords is that it is a vertically expressed CHORD. This is quite far from the truth. The melodic potential of Hexachords is unlocked when placed to good use in compositions (or improvisations) because they flow! Hexachords are moving lines that blend together creating seamless transitions between keys. Anything is Possible! Everything works!

The rest of the exercises in this book are advanced/further demonstrations of how the System works. As Bach constantly said and preached, TRUST YOUR EAR! It will tell you where to go. There are some compositional aspects of the system that we may dive into that explain how Classical Music works, but these melody etudes are open for all musicians to play. They will be in comfortable playing range for Saxophone players of all genres alike. The etudes are designed to demonstrate the connections in a way that will bring good absorption of the Hexachord System into finger and muscle memory.

As always, find a tempo that is easy and comfortable. There is no great reward in simply blazing through music too fast. Take your time. There will be occasions in the Etudes where I will purposefully stray from the original tonality, theme, etc; here is where you will learn the most. Remember... this system is universal. I am going to show you how and why, and I hope you’ll enjoy working with these pieces. It may surprise you to see what you can pick up in the moments you least expect...

4. ETUDES

No. 1 and No. 2 are simple introduction pieces written from a place of love in my heart. There is nothing overly complex about them and should be played in a sweet manner.
Etude No. 1

In the first etude, the use of the G# as a lean-to Servum utilizes the mirror of G/G# in the passing tonality of E major, moving around the home key of C. At Double Bar 2, the Secondaria is presented — the Key of G.

Etude No. 2

At Symbol *, A major is being used as an approach to C Major.

Etude No. 3

Etude No. 3 is inspired by the famous “Toccata in D-“ Hexachord, or commonly known Präludium Hexachord. It’s melody, when played by any large church organ, is immediately “Aha”-ed by any horror fan, or simple Halloween passerby. Harmonically, this Minor piece also focuses on a key switch from D Minor to C Minor, which is a nice sound to know.

Etude No. 4

This piece pays tribute to many of the joyful wonders in music and the world. Compositionally, it is in a similar lyrical style of some of my earlier saxophone recordings (Suite No. 1, Reflections Suite) which plays on melodies and their interaction with time and sound — time being a solid click*, and the sound being the growth of the melody within its related brother and sister tracks. There is nothing overly complex with the harmonies in this piece, play it with joy.

*When practicing with a metronome or recording with a click track, always use a number that is a multiple of 11. There is a unique phenomenon in this prime number’s reaction in a time-based sine wave that is more balanced than a non-multiple of eleven. Similar to how old clocks often tick at 55 or 66 BPM, or when a musical number is performed, our bodies have a natural tendency to find such a natural frequency...
**Etude No. 5**

No. 5 is light hearted and fun, like many of the classical etudes for Saxophone/Oboe. Enjoy running along with the high interval passages! They are quite pleasant once you get used to them. It is common in many musical studies to forget about larger interval leaps. In my time studying with Andy Middleton, we worked on large interval leaps over the entire saxophone’s range extensively, often to ensure a sense of technical mastery over any octave.

By symbol Omega, the passage uses a Hypo-Aeolian approach in C-Minor (G-A-B-C-D-E-F-G-Ab).

**Etude No. 6**

No. 6’s inspiration came from my local food store, Billa. They have a bell chime ringing at two different places that form a Hexachord with a harmonic sound that I can’t not sing a melody to each time I hear it! It comes from a B Maj/C Major combination*, and when the chimes go off at different times it creates such a wonderful chorale of sound. Inspiration is everywhere!

*A Hexachord with two major triads separated by a half step makes a beautiful “Harmonic” Minor or major sound. This particular combination is an essential part of Klezmer music and it’s Hexachordal combinations and inflections.

**Etude No. 7**

Walter Ernst Haberl plays a triplet-based trill warm-up over any major chord as soon as he sits at the keyboard. This piece also utilizes the “Employ & Erase” concept as described earlier. This one’s for you, my friend.

I thought it’d be enjoyable to switch between 2/4 and 6/8 in a few places. There is a small styling difference in phrasing when switching back and forth with the two time signatures! Look closely, follow what you think it should sound like, and you can make no mistakes.
Etude No. 8

This melody is one that came to me by pleasant surprise. The concept is to play the melody using multi-phonics, in this case by playing the overtones from the written note below. There is a space in-between certain intervals in the overtone series that can be played simultaneously with each other. Take the time necessary to find these notes before rushing into this piece. As with all publications or recordings, I believe that everything should have a closing reflection piece that brings the work to a whole. There is a beauty in ending things with a smile.

AFTERTHOUGHT/ACKNOWLEDGEMENTS

I cannot call the end of this paper a conclusion, because the music is self-conclusive. It is not an Epilogue or and Outro because it is not a closed story. This thesis is not being challenged, so there is no point to claim or persuade in defense. It is simply the beginning of what I intend to continue sharing with others throughout my career. It is, therefore, why I would like to add these afterthoughts to sum together this work as a whole.

The Hexachord System should not be forced upon any musician in any way, shape, or form. Music is a divine form of self-expression, and individuals have their own path to forge a new meaning for themselves. I understand clearly why the Hexachord System fell victim to the forgetting powers of time. The driving power behind the music, specifically in this case the Hexachord System, is that it in and of itself it is powerless; the user and protector decides its fate. It is not a long pedagogical approach that can be broken into multiple years of classes, split, flipped, and twisted around itself to form some sort of avant-garde creation! Any institutionalized system is inherently intimidated by an intangible, abstract concept with freedom as its only construct. That is exactly why I personally will not force the further development for what I believe is right—it is not for me to choose how others would like to think or create. I can only do my part in encouraging a space for this concept to flourish and grow, and let others seek its knowledge. This applies to more than just music; there is always so much to learn from something so small.
Above all, as I come to the end of this time studying in Vienna, and as I bring this paper to a close, I would like to acknowledge and give thanks to a few people that this work would not have been possible otherwise. First, to tip my hat to Thomas Huber, who helped make my voice not only come through on the saxophone, but who helped me find the joy in studying the great instrument once again. To Dr. Edwin Vanecek, for guiding the thoughts from my head onto paper, and getting everything ready in time. To Walter E. Haberl, whose insight into the entirety of music and the realm of Hexachords made it all worth it, and to the beautiful love of my life, Didi. I love you always and forever.
Appendix

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About the Author:

Beginning his first formal music studies at the age of 3 years old, Jake Goldwasser has always found himself working with the music inside his mind. By the age of 13, Jake began performing semi-professionally on both saxophone and piano. At 16, he was recording on full-length albums as a sideman for various Atlanta artists. After graduating High School, he took on the chance to explore the musical world in Europe, gaining the distinction of being the first and youngest accepted American student to the Musik und Kunst Universität der Stadt Wien’s prestigious Bachelor’s performance program. Since then, he has released 5 albums under his own name, while continuously working with other artists to help produce and bring them to finding their own unique sounds. This first written work, “An Introduction on the Universal Applications of Hexachords in Saxophone Performance and Technique” will act as an orientation for upcoming compositions and career directions as he continues his musical journey in America, where he will return upon graduation from the MUK. For more information, or to contact Jake at anytime, visit www.jakegoldwasser.com.

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Born January 15, 1997 in Atlanta, GA
2003-2015 Attended The Galloway School (Kindergarten—12th Grade)
2011 Performed in Carnegie Hall with National Youth Choir
2012 Participated in the Interlochen Summer Intensive Arts Camp (Jazz Program)
2013 Performed at the Atlanta Jazz Festival with the Rialto Youth Jazz Orchestra
2015-2019 Bachelor’s Degree at the Musik und Kunst Universität der Stadt Wien
2016 First solo album “Beginnings: Vienna” Released
2016 Second solo album “Sketches: Amsterdam” Released
2017 Third full album “Suite No.1 for Saxophone” Released
2017 Fourth full album “Conversations: The Reflection Suite” Released
2017 Fifth full album, First Solo Piano album “Memories” Released
2018 “Untitled” EP for Synthesizer Released